Algebra Isn't Hard
or
Demystifying Math: A Gentle Approach to Algebra

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Contents

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Introduction

Algebra Isn't Hard

Math isn't hard. No really. It is often explained in ways that are hard to understand. There are three common problems. First, many Math teachers forget that Math has its own language. Denominators and factors and quotients. They merrily speak a language you don't really understand, but can never quite understand why you are so confused. Second, they often explain a mathematical rule, give a few examples, then never (explicitly) mention it again. Of course the rule comes up over and over, but often students don't recognize non-trivial examples of the rule based on the trivial examples they were given when they were "on that chapter." Finally, Math teachers often don't explain how the rules inter-relate. Before "new math" teachers did proofs on the board, and students did proofs in their homework. Students were essentially required to understand the interrelation of the rules to progress. While doing proofs may have been taken to the point of absurdity, the contemporary phobia of proofs is no better. This relates to language as well. The difference between a denominator and a divisor is purely semantic. Without that knowledge the topic of fractions is harder than it need be.

This book will begin with topics that you already know, but the emphasis will be on concepts that were probably glossed over in your previous Math classes. We will build a foundation that will easily support your efforts to learn basic Algebra. You may be tempted to skip or "scan" this material. I assure you that this is a recipe for having a "hard" time with all of the subsequent material. Reading this material attentively, studying it, and learning it is a recipe for an easy time for the rest of this book, and a fantastic foundation for all your future Math study.

Structure of This Text

This text is structured to give students the maximum opportunity for success. Most chapters begin with the same two sections; Vocabulary and Notation. Familiarize yourself with the contents of these sections before reading the rest of the chapter. Refer to them while reading the rest of the chapter. You'll be amazed how much easier Math is when you speak the language!

The chapters are laid out in a structure that the author believes to be logical and conducive to success. Some trade-offs, however, have been made in order to keep related material together. This text is intended for a wide range of audiences, and some may find it beneficial to skip around a bit.

Goals of this Text

This text is written with two ends in mind. First, to convey the concepts of Algebra in an easily comprehensible (and maybe fun!) way. The second is to break down some of the fear and loathing that many students feel toward Math.

About this Edition

This is an "Alpha" version of the book. Meaning that it is incomplete and has not been thoroughly proofread. Attempts to learn Algebra from this book alone are doomed. If you are a student of Algebra I hope you are using this book in conjunction with a "real" Algebra text. If not, you must be proofreading. Thanks!

We used LyX, a Free, WYSIWYM (What You See Is What You Mean) document preparation system, to prepare the first several chapters of this book. LyX is something like a word processor, except that instead of explicitly telling it how to format a document the user selects a type of document and indicates the *meaning* of text while producing the document. For example, instead of inserting a "hard" page break and switching to a big font at the beginning of each chapter, I just type the name of the chapter and select "chapter" from the menu. This has innumerable advantages. For example, inserting a chapter causes a recalculation of the chapter numbers and re-generation of the table of contents. The bottom line is; less time spent formatting means more time for writing.

We abandoned LyX because it produces valid but unmanageable LaTeX output. We now compose in pure LaTeX, which makes managing the book much easier. If you want to make a contribution and you don't know LaTeX, we will accept your submission or corrections in plain ASCII (or UTF-8) text format. (Please note that "bare newlines" are the One True Line Ending.)

See http://www.lyx.org for more information about LyX. See http://www.gnu.org for more information about Free Software.

Part I

Fundamentals of Algebra

Chapter 1

Arithmetic

Arithmetic? Why should we cover something you mastered years ago? Two reasons. First, you know how to do it, but you probably don't really understand it in the way that will make Algebra easy. Second, one of the "hardest" things about Algebra is the sudden appearance of symbolic math. Symbolic math isn't really hard, but it is scary at first. By re-learning arithmetic symbolically, we ease into this scary topic with something you already know.

1.1 Vocabulary

Property

Notation Notation includes nearly everything you write in Math, from the meaning contained in how numbers are physically arranged to the strange symbols we resort to in expressing complex ideas. It may all look like Greek to you, but really only about a third of it is!

Implicit Implicit is the opposite of explicit. It comes from the same root as *imply*. The *notation* $4x$ is an example of implicit multiplication; it means that we have four times x.

1.2 Notation

1.2.1 Algebra Style Multiplication Notation

There is really only one bit of notation in this chapter that there is any real chance you are not familiar with. Multiplication is such a fundamental operation in Algebra that it is often *implicit*. Also, x is by far the most common variable. To avoid confusion a dot "$\cdot$" is used in place of the familiar multiplication sign "$\times$." Implicit multiplication looks like $5x$ which means $5 \times x$, or like $9(x + 3)$ which means "add three to the variable "x" *then* multiply by nine."

We have used dots everywhere throughout the first three chapters, because it is critical that you understand them. After the third chapter we will start using implicit notation the way it is normally used in Algebra.

1.2.2 Notation Introduced in this Chapter

For completeness we list the notation used in this chapter.

$=$ The equal sign. This symbol is a statement of fact. For example $x = 5$ means that the symbol x has the value 5.

$+$ Add e.g. $4 + 3 = 7$.

$-$ Subtract e.g. $4 - 3 = 1$.

$\times$, $\cdot$, or implicit Multiply e.g. $4 \times 3 = 12$, $4 \cdot 3 = 12$, or $4(3) = 12$.

$\div$ or / We do not divide! More in section 1.6.

1.3 Addition

Addition has four rules (or *properties*).

1.3.1 The Commutative Property of Addition

The commutative property of addition says that when adding numbers, the order doesn't matter. This is normally symbolized as $a + b = b + a$. For example $1 + 2 = 3$ and $2 + 1 = 3$. We'll find this convenient later for grouping things together in *algebraic expressions*. What do a and b really mean? They can be real numbers, which are all the numbers you know so far, including positive, negative, fractional, decimal (including irrational decimals, like π).

They an also be *variables* or algebraic expressions. So, you know for a fact that $x + r = r + x$ given that r is a real number and x is a variable, even if you don't know what that means! This will come in handy later when we can re-write stuff in an easier to read (and manipulate) way.

1.3.2 The Associative Property of Addition

The associative property of addition says that when we add more than two numbers together, we can group them any way we want. This is normally symbolized as $(a + b) + c = a + (b + c)$. Notice that we didn't change the order that they are in, just the order that we add them up in. This is a hard way of saying that the commutative property works for more than two numbers. What is the difference between saying $(a+b)+c = a+(b+c)$ and saying $(b+c)+a = a+(b+c)$? That's the order you are going to add them in, isn't it?

The important thing is this: you can add any number of things (numbers, variables, expressions) in any order you like, or that you find convenient.

1.3.3 The Identity and Inverse Properties of addition

The identity and inverse properties of addition are just a couple of obvious things written down so that we can rely on them to prove other things later. The identity property says that $a + 0 = a$. For example $\frac{\Theta\beta}{Rx} + 97q + 0 = \frac{\Theta\beta}{Rx} + 97q$. It doesn't matter what that other junk is (it's actually just junk), the point is that you can count on this rule. The inverse property has to do with the whole concept of negative numbers and subtraction, which we come to presently.

1.4 Subtraction and Negative Numbers

In easy Math, you can only have subtraction or negative numbers. You can't have both. (We're going to pick negative numbers.)

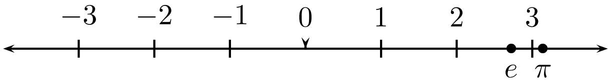

Figure 1.1: The arrow heads at the ends mean that the line goes on forever.

1.4.1 The Number Line

You may have seen a number line before. The number line is a *line* that is made up of an *infinite* number of points, each one describing its own distance from zero. So 1 is 1 unit away from zero. Zero is zero units away from zero, which works out nicely, don't you think? The number line looks like this:

The number line allows us to depict addition visually. $2 + 1 = 3$ means that if we travel the amount of distance from 0 to 1, *starting* at 2, we will end up at 3. The number line will also let us visually depict number theory in chapter 6. It is also the basis for graphing, which we will introduce in chapter 10.

1.4.2 Subtraction

Subtraction is the same concept as addition, except the second number is taken to be its opposite. $2 - 1 = 1$ means that if we travel the amount of distance from 0 to -1, *starting* at 2 we end up at 1.

So, there really isn't any such thing as subtraction, is there? The whole idea of subtraction is taught so that Math teachers don't have to teach first graders about the number line. So forget everything you know about subtraction, just add numbers. If some of them are negative that's fine.

My Algebra book spent a bunch of time explaining the rules of subtraction. Guess what, they are exactly the same as the rules of addition *since it is the exact same thing*.

1.4.3 The Inverse Property of Addition

The inverse property of addition says that $a + (-a) = 0$. In other words, every number is the same distance away from zero as the number that is the same distance away from zero in the opposite direction. Whew. You are probably the same height as all the other people in your class that are the same height as you . . . what a silly thing to say. But, as always, this rule is needed in future proofs.

1.5 Multiplication

1.5.1 The Commutative Property of Multiplication

The commutative property of multiplication says that when multiplying numbers, the order doesn't matter. This is normally symbolized as $a \cdot b = b \cdot a$. For example $4 \cdot 3 = 12 = 3 \cdot 4$.

1.5.2 The Associative Property of Multiplication

The associative property of multiplication says that when we multiply more than two numbers together, we can group them any way we want. This is normally symbolized as $a \cdot (b \cdot c) = (a \cdot b) \cdot c$.

Don't let the similarity of the rules for addition and multiplication fool you, though. The rules are the same, *but they don't mix*. More in chapter 3.

1.5.3 The Multiplicative Identity

We saw in section 1.3.3 that the additive identity of addition involves zero. Multiplying by zero, however, yields zero. The multiplicative identity is $a \cdot 1 = a$.

1.5.4 The Distributive Property of Multiplication

Multiplication has another wrinkle called the distributive property. It says that you can multiply before adding (in apparent contradiction of order of operations) *as long as* you multiply *every* term. The distributive property is normally symbolized as $a \cdot (b+c) = a \cdot b + a \cdot c$. For example $5 \cdot (6+2) = 30+10 = 40 = 5 \cdot 8$.

1.6 Division "and" Fractions

In easy Math you can either have division or fractions. You can't have both. We're going to pick fractions. I can almost hear you "Aww, I thought this was supposed to be easy . . . fractions are hard!" Fractions are important, and thinking in terms of fractions will help you in your Math education. Besides, they aren't really hard. Say goodbye to division!

1.6.1 The Idea

The fact is that dividing by a number and multiplying by its inverse are the same thing. Incidentally, a fraction is *a* number. It indicates a point on the number line *between* two whole numbers. Someone probably taught you to divide a fraction by another fraction by the "invert and multiply" method. The fact is that this is fundamentally how all division is done, but when dealing with whole numbers this inversion is implicit. For example $4 \div 2 = 4 \cdot \frac{1}{2} = \frac{4}{2} = 2$ (by the multiplicative identity and the "invert and multiply" rule). This example shows that the notation is even almost the same. The fact is that the symbol "$\div$" represents a fraction, as you can clearly see.

Long division, then, is a technique that is *handy* for whole numbers and decimals, but is a poor conceptual "definition" of division. When you think division, think invert and multiply.

1.6.2 Some Rules

1.6.3 Some Hints

1.6.4 What About Decimals?

We don't use decimals. Period (pun intended). We will see in Chapter 6 that decimals are a tool properly reserved for Science and Engineering. If your teacher lets you get away with using decimals he or she is doing you a disservice. Use fractions in Math. In Math we are only concerned with correct answers. Decimals, and particularly the decimals produced by a calculator, are implicitly estimates. From a Math point of view estimates are just as wrong as blank spaces and wild guesses.

There *are* a very few numbers that we use that can't be expressed as fractions. We have symbols for those numbers. We will encounter two such numbers in the course of this text: π and e.

1.7 What Does This All Mean?

1.8 Exercises

Chapter 2

More Arithmetic

2.1 Vocabulary

2.2 Notation

2.3 Multiplication as a Special Case of Addition

An Elementary School teacher may have explained multiplication to you as a special case of addition. This is a useful way of looking at it. You could expand $a \cdot b$ to "'b' 'a-s' added up." For example $3 \cdot 4$ means "four threes added up" or $3 + 3 + 3 + 3 = 12$ This works for all cases, even $a \cdot 0$, $a \cdot 1$, and $a \cdot (-1)$.

2.4 Exponentiation as a Special Case of Multiplication

The reason viewing multiplication as a special case of addition is useful is that the parallel holds as we progress to exponentiation. So a^b expands to "'b' 'a-s' multiplied together." This works for all cases, even a^0, a^1, and a^{-1}. You don't believe me? Okay, one at a time.

2.4.1 A Number Raised to the Zero Power

The rule is that any number (even zero) raised to the zero power is 1. A proof is provided in the appendix. A basic explanation that you can believe in without slogging through the proof follows.

> When you add, you start with an implicit zero. For example $(0) + 4 + 3 = 4 + 3$. This is the additive identity property at work. With multiplication, however, you start with an implicit *1*. As $(1) \cdot 4 \cdot 3 = 4 \cdot 3$. That is the multiplicative identity. So where we have, say, nine to the third power we have $9^3 = 1 \cdot 9 \cdot 9 \cdot 9$. Nine to the first power is $9^1 = (1) \cdot 9$. Then Nine to the zero is $(1) = 1$. Note that there is no nine here at all. So zero to the zero is $0^0 = (1) = 1$.

2.4.2 A Number Raised to the First Power

We saw above that $9^1 = (1) \cdot 9 = 9$. The fact is that any number written without an exponent has an implicit exponent of one. Because of the rules of order of operations we can only add, subtract, multiply, and divide numbers with the same exponent. In the vast majority of cases that exponent is one.

2.4.2.1 Multiply Numbers with the Same Base by Adding the Exponents

We can surmise from the above that we can re-write numbers with (and *only* with) the same base by adding their exponents. That is to say $a^b \cdot a^c \cdot a^d = a^{b+c+d}$. This is really just a form of multiplying. The reverse is a form of "factoring."

2.4.3 A Number Raised to the Negative First Power

We learned in 1.6 that division is the arithmetic inverse of multiplication. The negative sign is used to indicate the arithmetic inverse of addition. If we apply the fact that the negative sign means "do the inverse operation" with the idea that a fraction is the opposite of a product we get the affect of negative exponents. Specifically, $9^{-1} = \frac{1}{9^1} = \frac{1}{9}$. Using a different exponent will give us a better view of what is happening. $9^{-3} = \frac{1}{9^3} = \frac{1}{729}$.

> This effect is immensely useful in Science, particularly when the base 10 is used. For example $10^{-3} = \frac{1}{1000} = .001$. These powers of ten provide convenient multipliers to shift decimal places around. For example, .00000000000234 can be more meaningfully conveyed as 2.34×10^{-12}. This method is so useful, and so commonly used in Science that it is called "Scientific Notation."

2.5 Fractional Powers as the Arithmetic Inverse of Exponentiation

2.5.1 The $\frac{1}{2}$Power

2.5.2 The $\frac{1}{b}$Power

2.5.3 The $\frac{a}{b}$Power

2.5.4 Radicals

Radicals are a clumsy alternative notation to rational exponents. You are probably familiar with seeing the square root of 4, which we write as $4^{\frac{1}{2}}$ as $\sqrt{4}$. You may have also seen the cube root of 8 as $\sqrt[3]{8}$. This notation is associated with order of operations errors. It is included here in case you encounter it elsewhere (such as in a Science class).

2.6 Logarithms as the Algebraic Inverse of Exponentiation

See 11.4 "Inverse Functions" for more detailed discussion of inverse functions. When we say inverse function we mean that the answer becomes the question and the question becomes the answer. For example, in the expression $a^b = x$ the "question" is "what is a raised to the b power." The answer is "x." The inverse function would be $log_a x = b$ or "by what power must we raise "a" to obtain "x." The answer is "b." Many students find logarithms difficult. For now you can be successful if you learn the terminology and come to understand the relationships of the terms.

2.6.1 Simple Logarithms

2.6.2 Change of Base

2.7 Exercises

Chapter 3

Order of Operations

One of the nice side effects of the "easy" way we choose to view arithmetic is that order of operations is significantly simplified.

3.1 Vocabulary

3.2 Notation

(**and**) Parentheses are used to override order of operations rules.

[**and**] Square brackets are used in the same way as parentheses, but usually enclose larger groups consisting of at least one parenthesized group.

3.3 Order of Operations

What is the value of x given $3 + 4 \cdot 2 = x$? What about $4 \cdot 2 + 3 = x$ or $2 \cdot (3 + 4) = x$? In order to find the correct answers we must obey the rules of order of operations.

1. Outer brackets and parentheses.
2. Inner parentheses.
3. Exponents (and Logarithms)
4. Multiplication ("and" division).
5. Addition ("and" subtraction).

Applying these rules we see that $3 + 4 \cdot 2 = 11$, $4 \cdot 2 + 3 = 11$, and $2 \cdot (3 + 4) = 14$.

3.4 Exercises

Chapter 4

Simple Algebraic Operations

4.1 Vocabulary

4.2 Notation

= In the past you have seen the equal sign used as a question mark. In this chapter it is used as a period. An equal sign in Algebra is a statement of *fact*, though it still frequently implies a question.

4.3 Ordering Terms

4.4 Addition

4.5 Multiplication

4.6 Exercises

Chapter 5

Factoring

5.1 Vocabulary

5.2 Notation

5.3 Basic Factoring

Whereas division is the arithmetic inverse of multiplication, factoring is the algebraic inverse of multiplication.

5.3.1 What is Factoring?

Factoring is using the rules of arithmetic to re-write or simplify an expression to our advantage. Mainly this consists of exploiting the fact that the multiplicative identity is 1.

5.3.2 Caution in Factoring

One of the initial hurdles of Algebra is learning to rigorously apply the rules of order of operations and the properties of addition, multiplication, and exponents while factoring. For example:

$\frac{ax+b}{b} = ax$ WRONG!

Students seem to confuse the above with a case like:

$\frac{abx+b}{b} = \frac{b(ax+1)}{b} = ax$ True!

But even this case causes confusion:

$\frac{abx+b}{b} = \frac{b(ax)}{b} = ax$ WRONG!

If you are ever unsure that you have factored correctly, simply multiply your answer back out and see if you can get back to the original statement. If you do that with the incorrect example above you'll see that you've lost a b.

$\frac{b}{b} \cdot ax = \frac{abx}{b} = \ldots$ We can't get back to $\frac{abx+b}{b}$.

$\frac{b}{b} \cdot (ax+1) = \frac{b \cdot (ax+1)}{b} = \frac{abx+b}{b}$ That's it.

5.4 Special Factors

There are several "special" factors that occur often in Mathematics texts and rarely elsewhere, that are normally covered in Algebra. These are something of a parlor trick, but we would be remiss if we did not include them here.

Special Product	Special Factors	Example
$(a^2 - b^2)$	$(a+b)(a-b)$	$(x^2-4) = (x+2)(x-2)$
$(a^3 - b^3)$	$(a-b)(a^2+ab+b^2)$	$(x^3-8) = (x-2)(x^2+2x+4)$
$(a^3 + b^3)$	$(a+b)(a^2-ab+b^2)$	$(x^3+8) = (x+2)(x^2-2x+4)$
$(a^n - b^n)$	$(a-b)(a^{n-1}+ab+\cdots+b^{n-1})$	$(x^3-8) = (x-2)(x^2+2x+4)$
$(a^n + b^n)$	$(a+b)(a^{n-1}-ab+b^{n-1})$	$(x^3+8) = (x+2)(x^2-2x+4)$

5.5 Special Products

5.6 Exercises

Chapter 6

Number Theory

Number theory is a sizable branch of Mathematics. We only need some rudimentary theory to do Algebra.

6.1 Vocabulary

6.2 Notation

6.3 Counting (or Natural) Numbers

6.4 Whole Numbers

6.5 Integers

6.6 Rational Numbers

6.7 Irrational Numbers

6.8 Imaginary Numbers

6.9 Decimals

Decimals have no place in Mathematics. A brief explanation of decimals follows, but there is never a reason to use them in Math. An explanation follows because decimals *are* useful in Science and Engineering.

Decimals are a special piece of notation. They extend the decimal system (???) to include fractions (and irrational numbers). First, let us consider what 34 means. The "3" means three tens and the "4" means four ones. We can represent the meaning of the "places" in the decimal system as powers of ten (hence the name). We start counting for this purpose at zero. As we will see in section 2.4.1, any number raised to the zero power is one. So $10^0 = 1$. So the first place indicates "ones." Next we have $10^1 = 10$, so the next place is "tens." The familiar pattern of 100s, 1,000s, 10,000s continues.

What is the next whole number exponent below zero? Negative one. We will learn in section 2.4.3 that a number raised to the negative first power is the inverse of that number. So, $10^{-1} = \frac{1}{10}$. The places to the right of the decimals represent negative powers of ten, starting with -1. Therefore, 0.5 means $\frac{5}{10} = \frac{1}{2}$.

6.10 Prime Numbers

If a number has no positive, whole factors other than 1 and itself it is said to be *prime*. We need to recognize prime numbers so we know when to stop factoring. Primes have special uses in more advanced Mathematics, notably in *Cryptography*.

6.11 Exercises

Chapter 7

Proportions

Proportions give us an opportunity to try out some of the theory we have been discussing.

7.1 Vocabulary

7.2 Notation

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7.3 Ratios

7.4 Exercises

Chapter 8

Sets and Intervals

8.1 Vocabulary

8.2 Notation

$\cup$ Union - A set composed of all of the elements of two other sets. So $[1, 10] \cup [5, 15]$ is $[1, 15]$.

$\cap$ Intersection - A set comprised of all the *common* elements of two other sets. So $[1, 10] \cap [5, 15]$ is $[5, 10]$.

$\bigcirc$ Empty Set. For example $[1, 10] \cap [20, 30]$ is $\bigcirc$.

$\mathbb{R}$ The set of all Real numbers

ϵ Epsilon

[**or**] The end of a range in a set that *includes* the last element. Negative three is in the set $[-3, 4)$.

(**or**) The end of a range in a set that *excludes* the last element. Four is *not* in the set $[-3, 4)$, but $3\,\frac{999}{1000}$ is in the set.

{ **or** } Indicates interval notation.

8.3 Exercises

Chapter 9

Equality and Inequality

9.1 Vocabulary

9.2 Notation

=

<

>

$\leq$

$\geq$

9.3 Exercises

Chapter 10

Graphing Linear Equations

10.1 The Plane

Recall from 1.4.1 that the number line is a line that is made up of an infinite number of points, each one describing its own distance from zero. We know that we can find the value or values of x for a given value of y. So for $y = x$ we can generate the following table:

x	y
1	1
2	2
3	3
-1	-1
-2	-2
0	0

Part II

Functions and Graphs

10.2 Exercises

Chapter 11

Functions

11.1 Vocabulary

Domain All of the x values for which the function is defined.

Range All of the y values resulting in the domain.

Inverse Functions A pair of functions who's domain and range are exactly reversed.

11.2 Notation

$f(x) = x$ Read as "Eff of ex equals ex." This is exactly equivalent to $y = x$. Be aware, however that not all equations are functions. For example the equation of a circle of radius 1 and centered at the origin, $x^2 + y^2 = 1$, cannot be expressed as a (single) function.

$f^{-1}(x) = x$ The inverse function of f(x). In this case $f^{-1}(x) = f(x)$. Can you see why algebraically? How about graphically?

11.3 Functions as a Machine

It is often convenient to think of a function as a machine. Something goes in (x) and something comes out ($f(x)$). You could also think of it as a rule, or a game. You could read $f(x) = x^2$ as "I'll say a number and you give me its square. Ready?"

I say, "1."
You say, "1."
I say, "2."
You say, "4."
I say, "$47\frac{26}{37}$."
You say, "$2275\frac{750}{1369}$.

11.4 Inverse Functions

The inverse of a function means the function in which the domain and range are switched.

11.4.1 Cautions

11.5 Rational Functions

11.6 Functions of Functions

11.7 Exercises

Chapter 12

Graphing Linear Functions

12.1 Vocabulary

12.2 Notation

12.3 How do we Know a Function is Linear?

12.4 The Most Straightforward Method

12.5 The Standard Form of a Linear Equation.

The standard form of a linear equation is $y = mx + b$, where:

y	The variable that defines the vertical component of the equation.
m	The *slope* of the graph of the equation.
x	The variable that defines the horizontal component of the equation.
b	The y *intercept* of the graph of the equation.

Blah

12.6 Exercises

Chapter 13

More on Linear Functions

13.1 Vocabulary

13.2 Notation

x_1 Pronounced “Ex sub one.” A *subscript* indicates a specific instance or value of a variable. For example, if we are working with two ordered pairs we might call them (x_1, y_1) and (x_2, y_2) to keep them straight.

13.3 The Distance Formula

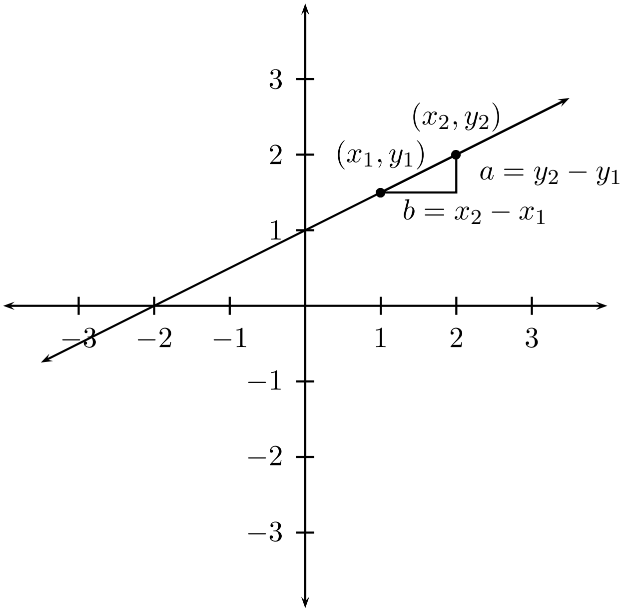

Figure 13.1: The Distance Formula: $d = \Big((x_2 - x_1) + (y_2 - y_2)\Big)^{\frac{1}{2}}$

$$\boxed{d = \Big((x_2 - x_1) + (y_2 - y_2)\Big)^{\frac{1}{2}}}$$

13.4 The Pythagorean Theorem

Many Algebra students are familiar with the Pythagorean Theorem. It states simply $a^2 + b^2 = c^2$ where b and a are the base and height of the triangle, respectively, and c is it's hypotenuse of a *right* triangle. [1]

13.5 The difference

13.6 Exercises

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[1] Incidentally, the Pythagorean theorem turns out to be a special "degenerate" case of the laws of sines and cosigns. These laws add a term to account for the variation from this relation that appears in acute and obtuse triangles. You learned, or will learn, these laws in Trigonometry.

Chapter 14

Systems of Equations

14.1 Vocabulary

14.2 Notation

14.3 Exercises

Chapter 15

Polynomials

Chapter 16

Quadratic Functions and Equations

16.1 Vocabulary

16.2 Notation

16.3 Quadratic Equations

16.4 Completing the Square

16.5 The Quadratic Formula

$$\boxed{x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}}$$

16.6 Deriving the Quadratic Formula

$ax^2 + bx + c = 0$	Start with the standard form of a quadratic equation.
$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$	Prepare to complete the square by dividing by the coefficient of x^2.
$x^2 + \frac{b}{a}x + (\frac{b}{2a})^2 = -\frac{c}{a} + (\frac{b}{2a})^2$	Add $(\frac{b}{2a})^2$ to both sides of the equation to complete the square.
$(x + \frac{b}{2a})^2 = -\frac{4ac}{4a^2} + \frac{b^2}{4a^2}$	Simplify both sides of the equation.
$(x + \frac{b}{2a})^2 = \frac{b^2-4ac}{4a^2}$	Simplify the right side further.
$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$	Take the square root of both sides, remembering to preserve the negative solution.
$x = -\frac{b}{2a} + \frac{\pm\sqrt{b^2-4ac}}{2a}$	Subtract $\frac{b}{2a}$ from both sides of the equation. Take the square root of the denominator.
$x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$	The Quadratic Formula

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Glossary

pi The ratio of the circumference of a circle to its diameter.

e The natural base.

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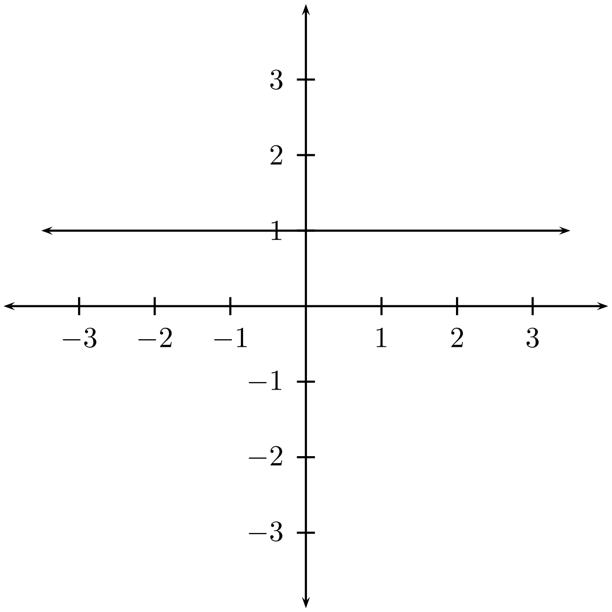

Figure E.1: The Constant Function $f(x) = 1$

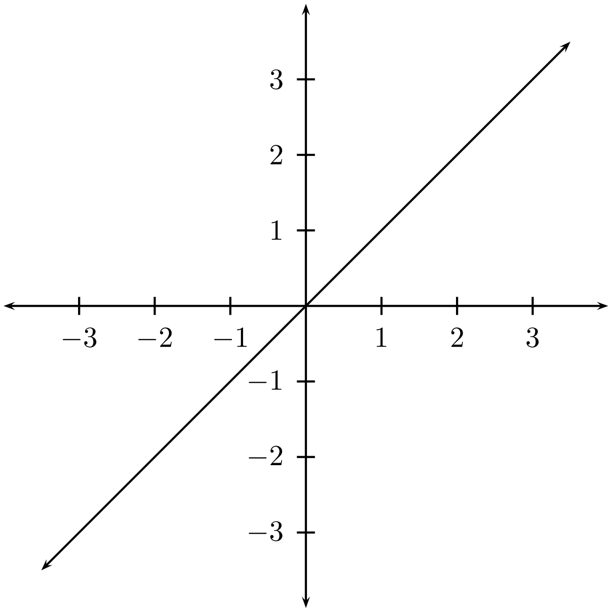

Figure E.2: The Identity Function: $f(x) = x$

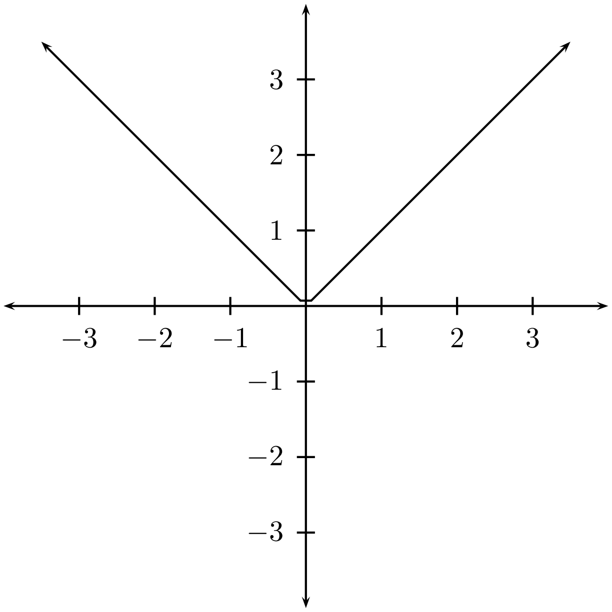

Figure E.3: The Absolute Value Function: $f(x) = |x|$

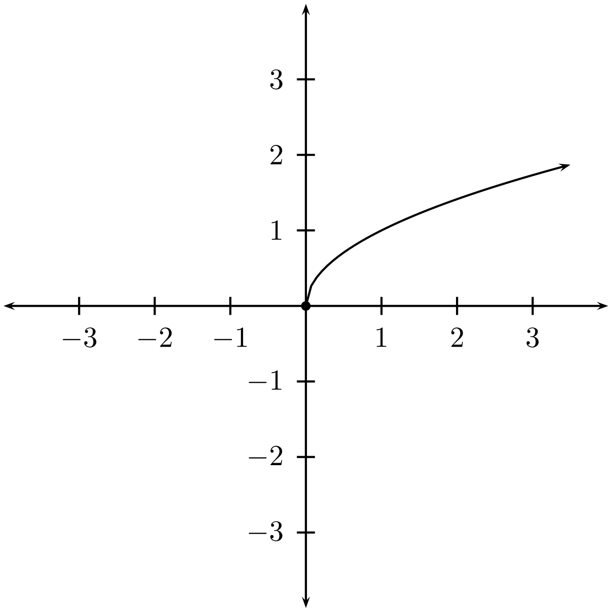

Figure E.4: The Square Root Function: $f(x) = x^{\frac{1}{2}}$

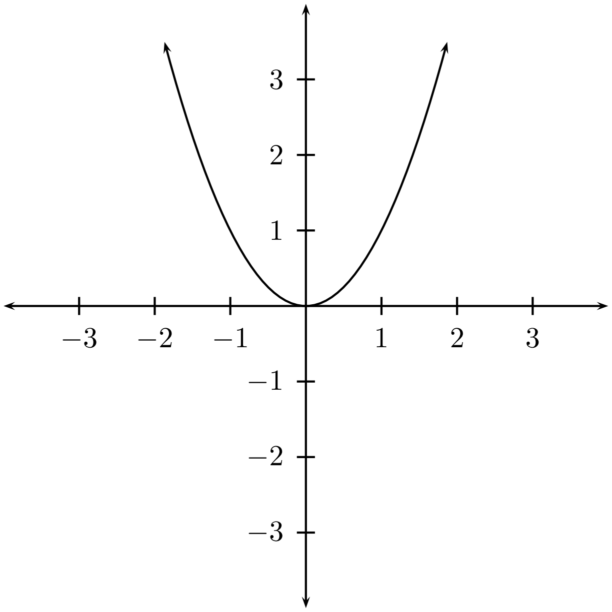

Figure E.5: The Square Function: $f(x) = x^2$

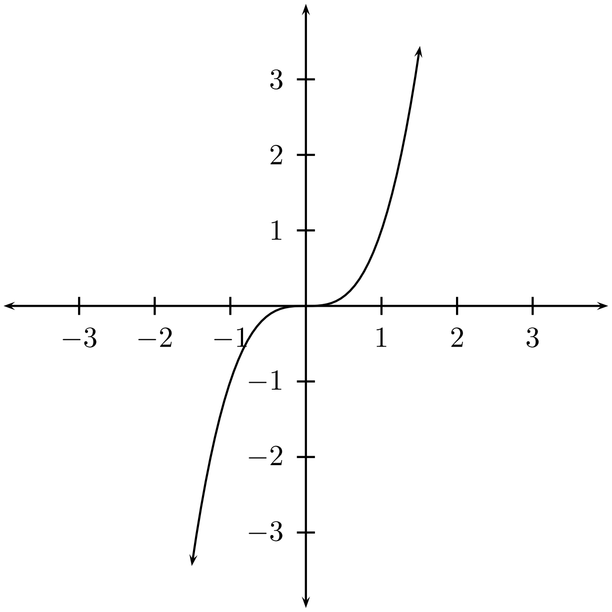

Figure E.6: The Cube Function: $f(x) = x^3$

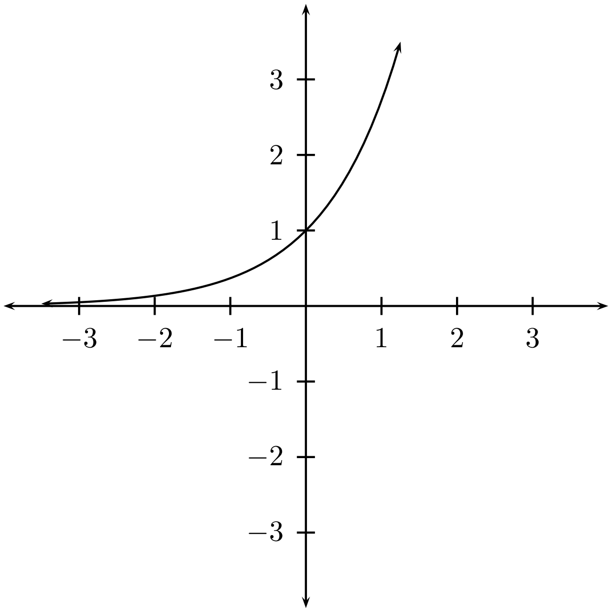

Figure E.7: The Exponential Function: $f(x) = e^x$

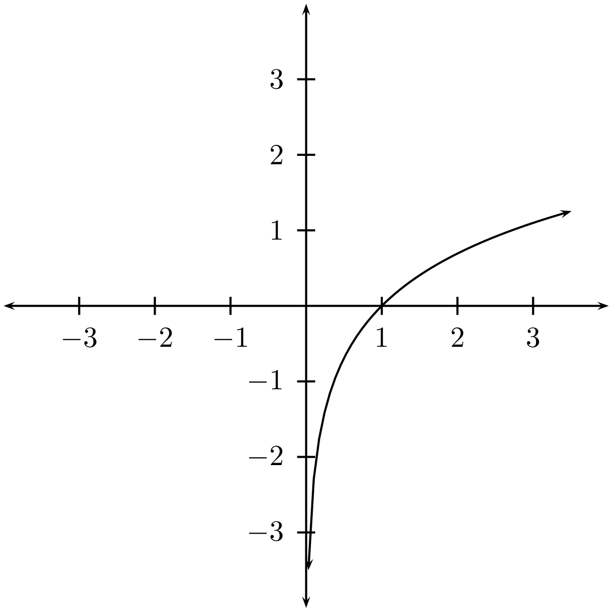

Figure E.8: The Logarithmic Function: $f(x) = ln(x)$